An analysis of Uniswap markets

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Outline

Measuring the price of assets

- Often, we need a way of measuring the price of an asset
- (Normally) easy: ask how much is someone willing to pay!
- In the traditional setting, this led to order books

Order book methods

- *Bid:* How much an agent is willing to pay for an asset
- *Ask:* How much an agent is willing to sell an asset for
- A trusted party keeps a record of all unfulfilled bids and asks
- When the highest bidder bids more than the lowest asker, the trade is executed
- The price of this trade is the 'current market price'

Disadvantages

- A trusted party keeps a record of all bids and asks

  Linear space requirement

- When the highest bidder bids more than the lowest asker [...]

  Price may update slowly, esp. with a small number of agents

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Automated Market Makers

Savage '71, Hanson '02

- Idea: use a (simple) formula to determine asset price
- Third-parties pool their assets (say A and B) into *reserves*
- Price set too low: agents purchase reserves at current price
- Price set too high: agents sell to reserves at current price

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- Price set too low: agents purchase reserves at current price
- Price set too high: agents sell to reserves at current price
- Using this idea, set price based on assets remaining in reserves
- *e.g.*, if too much of asset A remains, compared to asset B, decrease the price of A

Automated Market Maker examples

- Simplest example: fixed asset price at all reserve amounts
  i.e., a flat line
- Another example: reported price is ratio of two asset reserves
  This curve is Uniswap!

Uniswap (and constant product markets)

- Constant product markets (*e.g.*, Uniswap) is the family of curves whose reserves R_α, R_β must always satisfy:

$$R_\alpha R_\beta = k,$$

  for some constant k (no fees)

- In this case, we will assume that α and β are coins, though they can be any asset

- To satisfy this equation, the marginal price of asset β with respect to α is always

$$m_u = \frac{R_\beta}{R_\alpha}$$

Current situation DeFi

- People are using these markets!

5.	Uniswap	Ethereum	DEXes	$52.1M	0.7%

52.1M USD as of 11 AM yesterday (`defipulse.com`)

- Celo, *e.g.*, uses it as a price oracle
- So certainly worth analyzing!

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- An *arbitrageur* borrows an arbitrary amount of coin α or β but must pay it all back after their transaction (sound familiar?)
- The agent can then trade between two markets:
  1. Uniswap
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- Optimal strategy?

The arbitrage game

- Equivalent to the optimization problem

$$\begin{array}{ll} \text{maximize} & m_{\boldsymbol{p}}\Delta_\alpha - \Delta_\beta \\ \text{subject to} & (R_\alpha - \Delta_\alpha)(R_\beta + \Delta_\beta) = k. \end{array}$$

  Here, Δ_α is the amount of α traded and Δ_β is the amount of β traded.

- Optimal trade $(\Delta_\alpha^\star, \Delta_\beta^\star)$ always satisfies:

$$\frac{R_\beta + \Delta_\beta^\star}{R_\alpha - \Delta_\alpha^\star} = m_{\boldsymbol{p}},$$

  i.e., the new price equals to the market price!

More questions

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- Faced with these arbitrageurs, how much does manipulation cost?
- It is not hard to give a *per block* lower bound. Manipulating price to $(1+\varepsilon)m_p$ costs at least

$$C(\varepsilon) \geq KR_\alpha \min\{\varepsilon^2, \sqrt{\varepsilon}\},$$

and $K > 0$ a universal constant

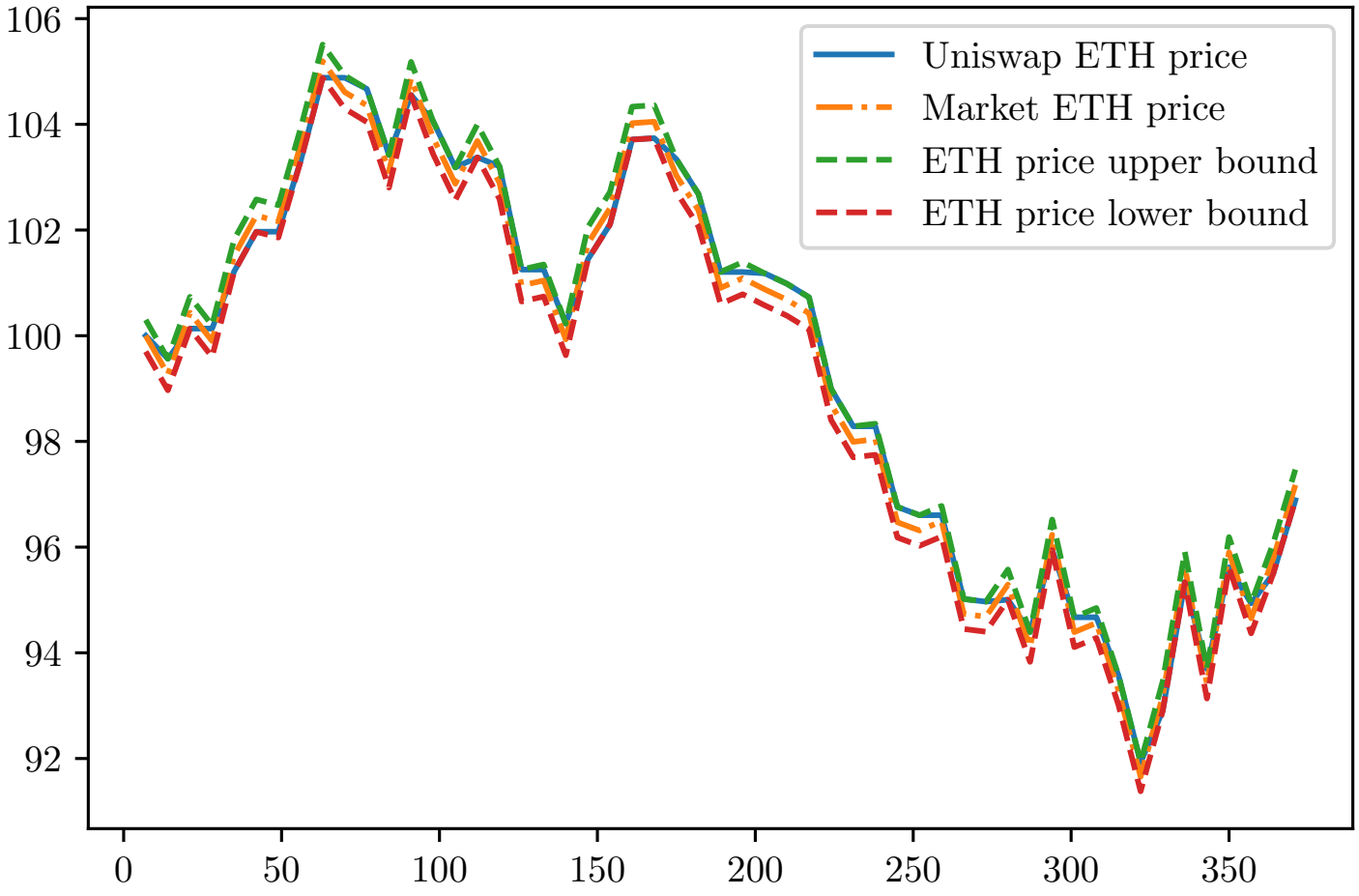
Important points

- Lower bound is zero if manipulation is within one transaction
- Manipulation over the short term is cheap
- As is manipulation where ε is small

Even more properties

- As expected, trading a fixed amount of desired coin will be cheaper as the reserves grow
- But liquidity providers will only (rationally) add coin to reserves if they believe m_p is driftless
- Additionally, Uniswap can never be drained of coin (*i.e.*, $R_\alpha + R_\beta \geq 2\sqrt{k}$ is always satisfied)

Simulations confirm these results:



Price in the case of no traders (with optimal arbitrage)

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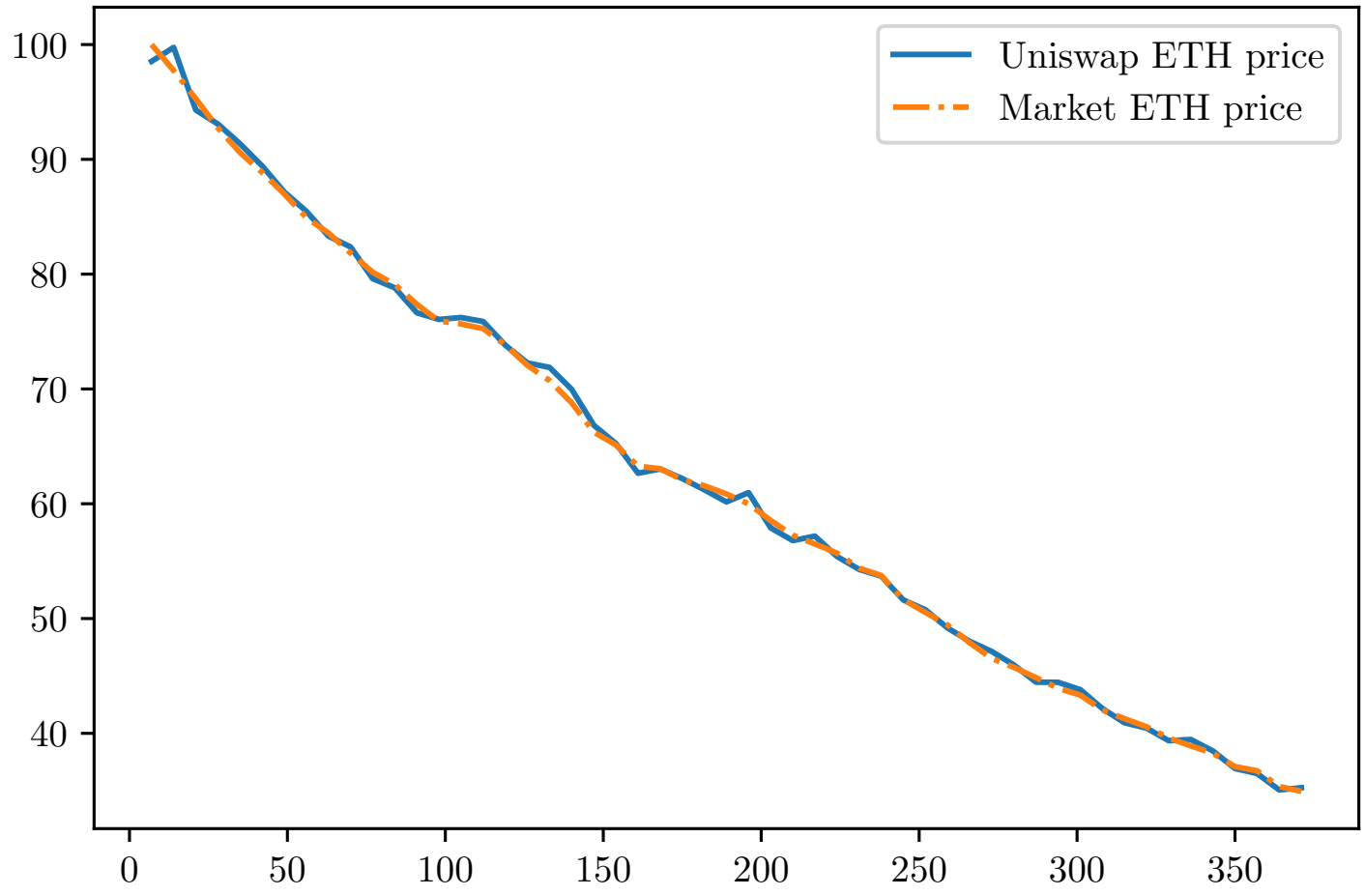
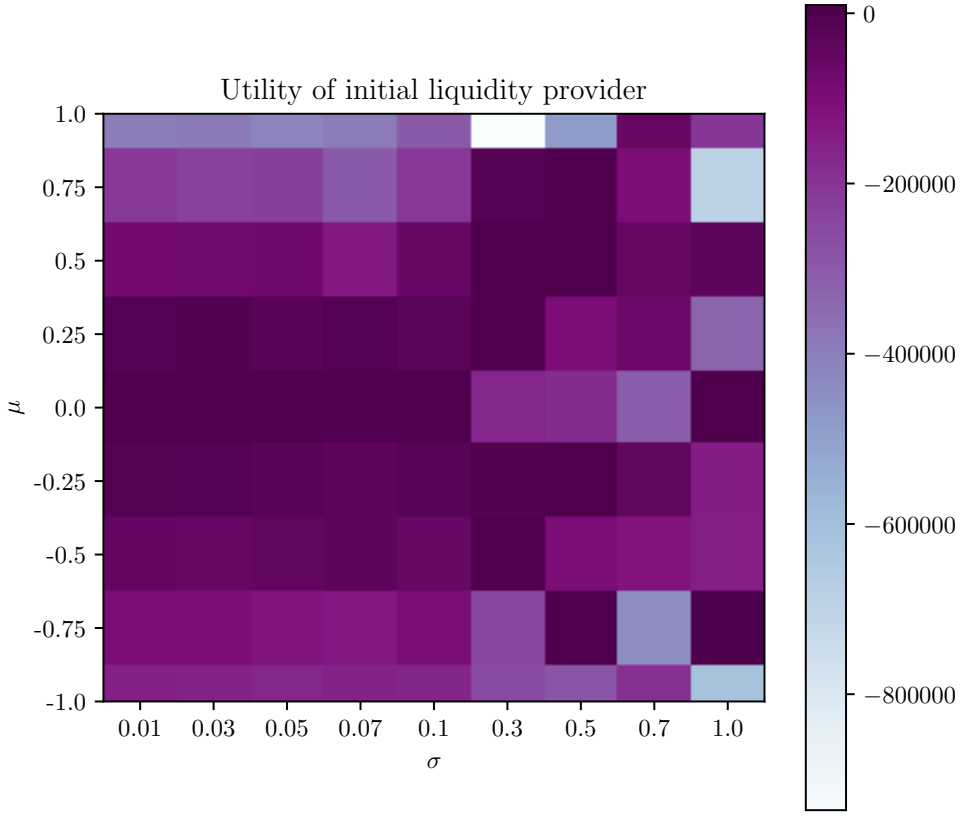


Price in the case of trading noise

Simulations confirm these results:



Initial LP utility vs HODL

Outline

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