Bounds on achievable performance via Lagrange duality

Guillermo Angeris[1,2*], Rahul Trivedi[1,2], Logan Su[2], Jelena Vučković[1,2], Stephen Boyd[1]

[1]Department of Electrical Engineering, Stanford University, Stanford, CA, United States
[2]E. L. Ginzton Laboratory, Stanford University, Stanford, CA, United States
*corresponding author, e-mail: `angeris@stanford.edu`

Abstract

While the problem of designing physical devices is computationally hard, heuristics such as inverse design are often successful in finding designs with very good performance. A natural question to ask is: how does the performance of these designs compare to the best possible performance? We show how to construct a convex (and therefore easily solvable) problem whose optimal value bounds the best achievable performance, for a given objective. We also provide a proof of the NP-hardness of physical design.

1. Introduction

Photonic inverse design has been an incredibly successful tool for finding devices with small footprint area whose performance is as good, and often better, than those found by traditional design methods [1]. Yet, many of the methods used for optimizing these designs are heuristics which do not guarantee that a globally optimal design has been found. Because of this, it is unclear if the designs found by these methods are close to being globally optimal, or if there exist much better designs that these methods are unable to find.

One way of answering this question is to give bounds on the best possible performance for a given metric: if a physical device achieves a performance close to this bound, then the physical device must also be close to globally optimal. There has been a great deal of recent work attempting to give bounds for particular performance metrics (see, *e.g.*, [2, 3]). In this talk, we provide a small extension to [4], which gives efficiently computable bounds on arbitrary separable objective functions, and show that it is computationally difficult to find a globally optimal design (*i.e.*, the design problem is NP-hard) for any reasonable choice of performance metric.

References

[1] S. Molesky, Z. Lin, A. Y. Piggott, W. Jin, J. Vučković, and A. W. Rodriguez, “Inverse design in nanophotonics,” *Nature Photonics*, vol. 12, no. 11, pp. 659–670, Nov. 2018.

[2] O. D. Miller, A. G. Polimeridis, M. T. Homer Reid, C. W. Hsu, B. G. DeLacy, J. D. Joannopoulos, M. Soljačić, and S. G. Johnson, “Fundamental limits to optical response in absorptive systems,” *Optics Express*, vol. 24, no. 4, p. 3329, Feb. 2016.

[3] S. Molesky, W. Jin, P. S. Venkataram, and A. W. Rodriguez, “Bounds on absorption and thermal radiation for arbitrary objects,” *Physical Review Letters*, vol. 123, no. 25, p. 257401, Dec. 2019.

[4] G. Angeris, J. Vučković, and S. Boyd, “Computational bounds for photonic design,” *ACS Photonics*, vol. 6, no. 5, pp. 1232–1239, May 2019.